PUPILS' OUTLINES FOR HOME STUDY

IN CONNECTION WITH SCHOOL WORK

ARITHMETIC, PART II

Percentage Mensuration

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PERCENTAGE

This deals with examples containing *per cents.*

Percent comes from *per* meaning *by* and *centum, hundred.* It is indicated by the *sign* % following the number.

Percents may be expressed three different ways by—

1. Sign..... % 2. Decimal..... .04

3. Fraction..... $\frac{4}{100}$ reduced to its lowest terms $\frac{1}{25}$

To express decimally a number with a % sign—omit the sign and place the decimal point two places to the left.

Example—Express $\frac{1}{2}$% as a decimal and as a fraction.

a. Percent meaning by the hundred, express two whole decimal places for the percent, but as there is no whole number given supply ciphers, as, .00$\frac{1}{2}$ or .005. Ans.

b. Expressing as a fraction take as a numerator the number given and make the denominator 100; as, $\frac{\frac{1}{2}}{100}$ simplifying $\frac{1}{2} \div \frac{100}{1} = \frac{1}{2} \times \frac{1}{100} = \frac{1}{200}$ Answer.

Reversing the operation, *a fraction being given, to express as a decimal and as per cent;* $\frac{1}{200}$.

Reduce to a decimal by dividing the numerator by the denominator as indicated by the fraction, annexing two ciphers as percent is *by the hundred.*

$$\begin{array}{r} .05 \text{ or } 5\%. \text{ Answer.} \\ 200)\overline{1.00} \\ \underline{1\ 00} \end{array}$$

Percentage

Base—means the whole and is the number of which some percent is found.

Rate—Rate percent is the number of hundredths which is taken of the base.

Percentage—is the number resulting after a certain percent has been taken of the base and is a *part* of the *whole.*

Formulas:

1. Base $\times$ $\frac{\text{Rate}}{100}$ = Percentage. 2. Base $+\left(\text{B} \times \frac{\text{R}}{100}\right)$ = Amount.

3. Base $-\left(\text{B} \times \frac{\text{R}}{100}\right)$ = Difference.

First Case—Finding a part of a quantity or percent of a number; as, What is 5% of $250?

The whole or number of which some part or percent is to be taken is $250—the rate 5% $\therefore$ $\text{B} \times \frac{\text{R}}{100} = \text{P}$. $\$250 \times \frac{\cancel{5}}{\cancel{100}} = \frac{25}{2} = \$12\frac{1}{2}$. Answer.

Second Case—Finding what part one number is of another or what percent one number is of another; as, What % is 12\frac{1}{2}$ of $250?

$\text{B} \times \frac{\text{R}}{100} = \text{P}$. $\therefore$ $\$250 \times ? = \$12\frac{1}{2}$

$12\frac{1}{2} \div 250 = \frac{\cancel{25}}{2} \times \frac{1}{\cancel{250}} = \frac{1}{20}$ or 5%

$$\begin{array}{r} .05 \text{ or } 5\%. \text{ Answer.} \\ 20)\overline{1.00} \\ \underline{1\ 00} \end{array}$$

Third Case—Finding a quantity when a part is given or finding a number when a percent of it is given; as, $12\frac{1}{2}$ is 5% of what number?

$\text{B} \times \frac{\text{R}}{100} = \text{P}$. $? \times \frac{5}{100} = \$12\frac{1}{2}$

$12\frac{1}{2} \div \frac{5}{100} = \frac{25}{\cancel{2}} \times \frac{\overset{10}{\cancel{100}}}{\cancel{5}} = \$250$. Answer.

375
P96P
v.2

Fourth Case—Finding a quantity when the quantity plus a part is given or finding a number when the number plus some percent of it is given; as,

A. What is \$250 increased by 5% of itself?

$$B + \left(B \times \frac{R}{100} \right) = \text{Amount.} \qquad \$250 + \left[250 \times \frac{5}{100} \right] = ?$$

(a.) $\$250 \times \frac{\overset{1}{\cancel{5}}}{\underset{20}{\cancel{100}}} = \frac{25}{2} = \$12\frac{1}{2}$. (b.) $\$250 + 12\frac{1}{2} = \$262\frac{1}{2}$. Answer.

B. What number increased by 5% of itself will give \$262½?

$$B + \left(B \times \frac{R}{100} \right) = \text{Amount.} \qquad ? + \left[? \times \frac{5}{100} \right] = 262\tfrac{1}{2}.$$

The base is the whole or 100%, hence

$$\frac{100}{100} + \left[\frac{100}{100} \times \frac{5}{100} \right] = \$262\tfrac{1}{2}. \qquad \frac{\cancel{100}}{\underset{20}{\cancel{100}}} \times \frac{\cancel{5}}{\cancel{100}} = \frac{1}{20}$$

Using the same kind of fraction—Base or the whole $\frac{20}{20} + \frac{1}{20} = \frac{21}{20}$ or $262\frac{1}{2}$.

$$\$262\tfrac{1}{2} \div \frac{21}{20} = \frac{\overset{175}{\cancel{525}}}{\cancel{2}} \times \frac{\overset{10}{\cancel{20}}}{\underset{7}{\cancel{21}}} = \frac{1750}{7} = \$250. \quad \text{Answer.}$$

C. \$250 being increased by a certain % of itself equals \$262½, find the rate?

$$B + \left(B \times \frac{R}{100} \right) = \text{Amt.} \quad \therefore \text{Amt.} - \text{Base} = \text{Percentage.} \qquad \$262\tfrac{1}{2} - \$250 = \$12\tfrac{1}{2}$$

$$B \times \frac{R}{100} = P. \qquad \$250 \times \frac{?}{100} = \$12\tfrac{1}{2} \qquad \$12\tfrac{1}{2} \div \$250 = \frac{25}{2} \times \frac{1}{\underset{10}{\cancel{250}}} = \frac{1}{20} \text{ or } 5\%. \text{ Ans.}$$

Fifth Case—Finding a quantity when the quantity minus a part is given, or finding a number when the number, minus some percent of it is given; as

A. What is \$250 decreased by 5% of itself?

$$B - \left(B \times \frac{R}{100} \right) = \text{Difference.} \qquad \$250 - \left[\$250 \times \frac{5}{100} \right] = ?$$

$\$\overset{5}{\cancel{250}} \times \frac{5}{\underset{2}{\cancel{100}}} = \frac{25}{2} = \$12\frac{1}{2}$. $\$250 - 12\frac{1}{2} = \$237\frac{1}{2}$. Answer.

B. What number decreased by 5% of itself will give \$237½?

$$B - \left(B \times \frac{R}{100} \right) = \text{Difference.} \qquad ? - \left[? \times \frac{5}{100} \right] = \$237\tfrac{1}{2}.$$

Base being the whole, may be represented by 100% or $\frac{100}{100}$ which is 1 or a whole, and solving the parenthesis we have $\frac{\overset{1}{\cancel{100}}}{\underset{20}{\cancel{100}}} \times \frac{\cancel{5}}{\underset{1}{\cancel{100}}} = \frac{1}{20}$

The base may be represented by $\frac{20}{20}$ or 1 whole, that fraction being easier to work with.

$\frac{20}{20} - \frac{1}{20} = \$237.50$. $\$237.50 \div \frac{19}{20} = \$237.50 \times \frac{20}{19} = \$250$. If \$237.50 is $\frac{19}{20}$, $\frac{1}{20}$ will be $\frac{1}{19}$ of \$237.50 or \$12.50, and the whole $\frac{20}{20}$ will be 20 times that or \$250.

C. What is the rate if \$250, decreased by a certain % of itself, equals \$237.50?

$$B - \left(B \times \frac{R}{100} \right) = \text{Difference.} \qquad \$250 - \left[250 \times \frac{?}{100} \right] = \$237.50.$$

Base — Difference = Percentage. \$250 — 237.50 = \$12.50 Percentage.

$$B \times \frac{R}{100} = P. \qquad \$250 \times \frac{?}{100} = \$12.50. \qquad \$12.50 \div \$250 = \frac{25}{2} \times \frac{1}{\cancel{250}\,10} = \frac{1}{20} = 5\% \text{ Ans.}$$

APPLICATIONS OF PERCENTAGE

In the application of percentage many new terms are introduced, but the principle of the work involved is more or less the same.

Percentage is based on multiplication, and remembering that having the product and one factor, the other is obtained by dividing the product by the given factor.

Multiplicand × Multiplier = Product.

The various names applied for the above terms are:

Percentage.....Base × Rate % = Percentage.

Profit and Loss.....Cost × Rate % Gain or Loss = Gain or Loss.

Commission..Sale, Collection, Purchase, or Money Involved × Rate % = Commission.

Taxes.....Property Value × Rate % = Tax.

Duties..... Cost × Rate % Duty = Duty Advalorem.

For Specific Duties deduct first the allowances for tare, if any, then get the duty on the remainder. Quantity × Rate = Specific Duty.

Insurance.....Number of Dollars Insured for × Rate % = Premium.

Trade Discount.....List Price × Rate % of Discount = Discount.

There is still another set of applications of percentage, where there are two factors in the multiplication, the element of time, as well as rate, having to be considered; as

Interest.....Principle × Rate % × $\frac{\text{Time in Days}}{360}$ = Interest.

True Discount..... { Principle or Present Worth } × Rate % × Time = Discount or Interest.

The above formulas may be used to find any missing term mentioned in them by inserting a question mark (?) or x for the missing term and supplying the other terms where necessary from the given example.

The element of **amount** or **difference** is sometimes introduced; as

Percentage.....
- B + { *Percentage* / B × R % } = Amount.
- B — { *Percentage* / B × R % } = Difference.

Profit or Loss.....
- Cost + { *Gain* / Cost × Rate % Gain } = Selling Price.
- Cost — { *Loss* / Cost × Rate % Loss } = Selling Price.

Commission
- { Sale, Collection or Money Involved } + { *Commission* / Sale, etc. × R % } + Charges if any = Entire Cost.
- Sale, etc. — { *Commission* / Sale, etc. × R % } = Net Proceeds.

Trade Discount.....List Price — { *Trade Discount* / List Price × R % } = Net Price.

Interest.....
- Prin. + { *Interest* / Prin. × R % × Time } = Amount.
- Prin. — { *Interest* / Prin. × R % × Time } = Difference.

Present Worth and True Discount } Present Worth + { *Discount* / Present Worth × R % × Time } = Debt or Amt.

Bank Discount.....
- Prin. × R % × Time = Bank Discount.
- Prin. — { *Bank Discount* / Prin. × R % × Time } = Net Proceeds.

Profit and Loss

This is the name given to examples dealing with the gain or loss in business transactions.

In showing the cost or selling price of goods, business men generally use a *key*, which is a system of lettering or symbols. Words made up of 10 different letters are used, each letter in turn standing for a figure; as,

black horse
12345 67890 } so if an article cost $596 it would be marked *ksh*.

For Formulas—see page 4.

Examples—1. A house was bought for $5,000 and sold at a gain of 8%. What was the gain?

Cost × Rate % = Gain. $\$50\not{0}\not{0} \times \frac{8}{1\not{0}\not{0}} = ?$ or $400. Answer.

2. Gained $400 by selling a house at a gain of 8%. What did it cost? Cost × Rate % = Gain.

$? \times \frac{8}{100} = \$400.$ $\$400 \div \frac{8}{100} = \$\overset{50}{\not{4}\not{0}\not{0}} \times \frac{100}{\not{8}} = \$5000.$ Answer.

3. A house costing $5000 was sold at a gain of $400. What was the gain %? Cost × Rate % = Gain. $\$5000 \times ? = \$400.$ $\$400 \div \$5000 = .08$ or 8%. Ans.

If this had been a *loss* % instead of gain it would have been worked the same. Substituting loss for gain in formula.

4. A house bought for $5000 was sold at a gain of 8%. What was the selling price?

$$\text{Cost} + \left(\text{Cost} \times \frac{\overset{\textit{Gain}}{\text{rate}}}{100}\right) = \text{S. P.}$$

$$\$5000 + \left(\$5000 \times \frac{8}{100}\right) = ? \qquad \$50\not{0}\not{0} \times \frac{8}{1\not{0}\not{0}} = \$400.$$

$5000 + 400 = $5400. Selling Price. Answer.

5. A man received $5400 for a house sold at a gain of 8%. What did it cost him?

$$\text{C} + \left(\text{C} \times \frac{\text{R}}{100}\right) = \text{S. P.}$$

$? + \left(? \times \frac{8}{100}\right) = \$5400.$ Cost is represented by 100% or $\frac{100}{100}$, the whole.

$$\frac{100}{100} + \left(\frac{100}{100} \times \frac{8}{100}\right) = \$5400.$$

(a.) $\frac{\not{1}\not{0}\not{0}}{\not{1}\not{0}\not{0}} \times \frac{\overset{2}{\not{8}}}{\underset{1}{\not{1}\not{0}\not{0}}} = \frac{2}{25}$ (b.) $\frac{100}{100}$ or $\frac{25}{25} + \frac{2}{25} = \$5400.$

$\$5400 \div \frac{27}{25} = \$\overset{200}{\not{5}\not{4}\not{0}\not{0}} \div \frac{25}{\not{2}\not{7}} = \$5000.$ Answer.

6. A man sold a house for $5400 which cost him $5000. What was his gain %?

$$\text{Cost} + \left\{\text{Cost} \times \frac{\overset{\textit{Gain}}{\text{R}}}{100}\right\} = \text{Amt.} \qquad \$5000 + \left(\$5000 \times \frac{?}{100}\right) = \$5400.$$

Amount — Cost = Gain. $5400 — 5000 = 400.

$\text{Cost} \times \frac{\text{R}}{100} = \text{Gain.}$ $5000 \times \frac{?}{100} = 400$ ∴

$400 \div 5000 = \overset{2}{\not{4}\not{0}\not{0}} \times \frac{1}{\underset{25}{\not{5}\not{0}\not{0}\not{0}}} = \frac{2}{25} = .08$ or 8%. Answer.

Had the above three examples indicated a loss instead of a gain, we would have used $\text{Cost} - \left(\text{Cost} \times \frac{\text{loss rate}}{100}\right) = \text{Difference}$, substituted the values and solved same as above.

Commission

This deals with examples where one person, an agent or broker, does business for another for which he receives a commission which is a certain percent of the money involved, that which has been collected or spent.

The **money collected** or **spent** corresponds to the **base.** The **rate of commission** corresponds to the **rate** %. The **commission** corresponds to the **percentage.**

The **entire cost** is the money Collected or Spent, plus the Commission, and other charges, if any, and corresponds to the **amount.**

The **net proceeds** is the remainder after the Commission and other expenses, if any, have been paid or deducted from the Money Collected or to be Spent and corresponds to the **difference.** Commission is given only on *Money Collected or Spent*, not on charges.

Example—1. An agent sold goods for $5250, his rate of commission being $2\frac{1}{2}$%. What commission did he receive?

$$\text{Money Collected} \times \frac{R}{100} = \text{Com.} \qquad \$5250 \times \frac{2\frac{1}{2}}{100} = ?$$

$$\$5250 \times \frac{5}{2} \times \frac{1}{100} = \frac{525}{4} = \$131\tfrac{1}{4}. \quad \text{Answer.}$$

2. An agent received $131\frac{1}{4}$ for selling goods at a commission of $2\frac{1}{2}$%. What were the goods sold for?

$$\text{Money Collected} \times \frac{R}{100} = \text{Com.} \qquad ? \times \frac{2\frac{1}{2}}{100} = \$131\tfrac{1}{4}.$$

$$\frac{5}{2} \div \frac{100}{1} = \frac{5}{2} \times \frac{1}{100} = \frac{1}{40} \qquad ? \times \frac{1}{40} = \$131\tfrac{1}{4}.$$

$$\frac{\$525}{4} \div \frac{1}{40} = \frac{525}{4} \times \frac{40}{1} = \$5250. \quad \text{Answer.}$$

3. An agent sold goods for $5250 and received $131.25 commission; what was his rate of commission?

$$\text{Money Collected} \times \frac{R}{100} = \text{Com.} \qquad \$5250 \times ? = \$131.25$$

$\$131.25 \div \$5250 = .02\frac{1}{2}$ or $2\frac{1}{2}$%. Answer.

4. An agent was to receive $2\frac{1}{2}$% commission for buying goods for $5250. What was the amount spent by the employer?

$$\text{Money Spent} + \left\{ \text{Money Spent} \times \frac{R}{100} \right\} = \text{Entire Cost.} \qquad \$5250 + \left(\$5250 \times \frac{2\frac{1}{2}}{100}\right) = ?$$

(a.) $5250 \times \frac{2\frac{1}{2}}{100} = \$131.25$ (b.) $\$5250 + \$131.25 = \$5381.25$ Answer.

5. An agent received $5381.25 to be invested in goods. After retaining his commission of $2\frac{1}{2}$% what money was spent for the goods?

$$\text{Money Spent} + \left\{ \text{Money Spent} \times \frac{R}{100} \right\} = \text{Entire Cost.} \qquad ? + \left(? \times \frac{2\frac{1}{2}}{100}\right) = \$5381.25$$

(The money spent is equal to 100% or $\frac{100}{100}$, the whole.) $\frac{100}{100} \times \frac{5}{2} \times \frac{1}{100} = \frac{1}{40}$

or for convenience $\frac{40}{40} + \frac{1}{40} = \$5381.25$ $\$5381.25 \div \frac{41}{40} = \$5381.25 \times \frac{40}{41} = \$5250$. Ans.

6. An agent drew $5381.25 for which he purchased goods for $5250; what was his rate of commission?

$$\text{Money Spent} + \left\{ \text{Money Spent} \times \frac{R}{100} \right\} = \text{Entire Cost.}$$

$$\$5250 + \left(\$5250 \times \frac{?}{100}\right) = \$5381.25$$

Entire Cost—Money Spent = Commission, $\$5381.25 - \$5250 = \$131.25$ Com.

$$\text{Money Spent} \times \frac{R}{100} = \text{Com.} \qquad 5250 \times \frac{?}{100} = \$131.25$$

$\$131.25 \div 5250 = .02\frac{1}{2}$ or $2\frac{1}{2}$%. Answer.

7. An agent collects $3500 on a commission of 2%. How much should he remit to his employer?

$$\text{Money Collected} - \left\{ \text{Money Collected} \times \frac{\text{R}}{100} \right\} = \text{Net Proceeds.}$$

Commission

$$\$3500 - \left(\$3500 \times \frac{2}{100}\right) = ?$$

(a.) $\$3500 \times \frac{2}{100} = \$70.$ (b.) $\$3500 - \$70 = \$3430.$ Answer.

8. A merchant received $3430 after the agent had made his collection and charged 2% commission. How much was collected?

$$\text{Money Collected} - \left\{ \text{Money Collected} \times \frac{\text{R}}{100} \right\} = \text{Net Proceeds.}$$

Commission

$$? - \left(? \times \frac{2}{100} \right) = \$3430.$$

$\frac{100}{100} \times \frac{2}{100} = \frac{1}{50}$ $\frac{50}{50} - \frac{1}{50} = \frac{49}{50}$ or $3430

$$\$3430 \div \frac{49}{50} = \$3430 \times \frac{50}{49} = \$3500. \text{ Answer.}$$

9. A merchant received $3430 after the agent had retained his commission, $70. What rate did he charge?

$$\text{Money Collected} - \left\{ \text{Money Collected} \times \frac{\text{R}}{100} \right\} = \text{Net Proceeds.}$$

Commission

Net Proceeds + Commission = Money Collected. $3430 + $70 = $3500.

$$\text{Money Collected} \times \frac{\text{R}}{100} = \text{Commission.} \qquad \$3500 \times \frac{?}{100} = \$70.$$

$70 ÷ $3500 = .02 or 2%. Answer.

10. An agent bought 40 boxes goods at $150 and paid $120 for transportation; he drew on me for $6300; what was the rate of commission?

$150 × 40 = $6000 Money spent for goods.
120 Money spent for transportation.

$6120 Money spent (total).

Money Remitted $6300 — $6120 Money Spent = $180 Commission.

$$\text{Money Spent} \times \frac{\text{R}}{100} = \text{Com.} \qquad \$6000 \times \frac{?}{100} = \$180. \qquad \$180 \div \$6000 = .03 = 3\%. \text{ Ans.}$$

Taxes—Duties or Customs

To meet the government expenses of the Nation, State, City, County, Town, etc., money is raised by taxation. **Taxes** are money paid on,

1. **Real Estate**—such as land and buildings. *Assessors* place a value on all real property known as the *assessed valuation*, which is usually less than the actual value. Upon this value the rate of taxation is charged.
2. **Personal Property**—such as money, bonds, etc. The value is usually given by the owner under oath.
3. **Persons**—a poll tax, usually a dollar for every male citizen over 21 years.
4. **Business, Corporations,** etc.
5. **Licenses**—right to enjoy some special privilege or carry on some special work.
6. **Imported Goods** and is then known as *tariff, customs* or *duties*. This is collected at the different custom houses located at each place where foreign vessels enter, and on the border between different countries.

A **tariff** is the list of duties on the different articles. There are two kinds of duty.

a. **Ad Valorem**—a certain percent of the value of the goods where bought.

b. **Specific Duty**—a certain sum charged according to quantity; so much for each bushel, each yard, etc., having nothing to do with value. In custom houses the long ton, 2240 lbs., is used. Sometimes articles are subject to both kinds of duties.

Taxes—Terms used: Assessed value of the property corresponds to the **base. Rate of taxation** to the **rate,** and is either expressed as a certain

a. *Percent* of the assessed value, or

b. Number *mills* on *each dollar* of the assessed value, or

c. Number *cents* on *each hundred* dollars of the assessed value.

So a tax of 3 mills on one dollar would be .003 or $\frac{3}{1000}$ or $\frac{3}{10}\%$.

Tax corresponds to the **percentage.**

Persons collecting taxes are sometimes paid for their services by the person paying the taxes in such cases it is not included in the amount of taxes to be raised. The fee is usually 1%.

Example 1. Property valued at $17,500 was taxed at the rate of 5 mills on the dollar. What was the tax paid by the owner, if his poll tax was $1?

Valuation × Rate = Taxes. $12,500 × .005 = $62.50

Property Tax + Poll Tax = Entire Tax. $62.50 + $1.00 = $63.50

2. What is the assessed value of property taxed $62.50 at the rate of 5 mills on the dollar?

Valuation × Rate = Taxes. ? × .005 = $62.50 $\quad \frac{\$62.50}{.005} = \$12500$

3. A tax of $7025 is to be assessed. The real property being valued at $600,000 and the personal at $375,000—there are 200 poll taxes at $1 each, what is the rate of taxation?

$1 × 200 = $200 money raised by the poll tax.

$7025 — 200 = $6825 money to be raised by taxing property.

600,000 + 375,000 = $975,000 value of property real and personal.

Valuation × R = Tax. $975,000 × ? = $6825

$\frac{6825}{975000}$ = .007 or 7 mills or $\frac{7}{10}\%$. Answer.

Customs and Duties

Tare is the allowance made for boxes, casks, etc., in which the goods are shipped. *Fractions less than ½ are rejected; more than ½ are counted another one.*

Example 1. What is the duty on 350 yards carpet costing $2.50 a yard, at 20% ad valorem? Value × Rate = Duty. $(\$2\frac{1}{2} \times 350) \times \frac{20}{100} =$?

$$\frac{\$\cancel{5}}{\cancel{2}} \times \frac{\overset{35}{\cancel{70}}}{\cancel{350}\,1} \times \frac{1}{\cancel{20}\,\cancel{100}\,\cancel{5}} = \$35. \quad \text{Answer.}$$

2. What is the specific duty at 3 cents a box on 80 boxes, each containing 25 lbs., tare 5 lbs. a box?

One box contains 25 lbs. so 80 boxes contain 80 times that amount or 2000 lbs., and 5 lbs. allowed on each of 80 boxes gives 400 lbs. tare—leaving 1600 lbs. on which duty is to be paid.

$25 \times 80 = 2000$ lbs.
$5 \times 80 = 400$ lbs.
1600 lbs.

Number lbs. × Rate = Duty. 1600 × .03 = $48.00 Answer.

Property Insurance

This is an agreement whereby an Insurance Company will pay for some specified loss.

Policy is the written agreement to pay a specified amount in case of loss. The amount specified is called the **face of the policy** and corresponds to the **base.** Policies are renewed *annually* or for *three* or *five years.* If a policy is taken for 3 or 5 years there is always a reduction made; for instance for 3 years—only the amount for two annual sums, instead of three, would be paid; and for 5 years—only three times the annual sum instead of five.

Premium is the money paid to the Insurance Company for undertaking the risk and corresponds to the **percentage.** This is paid in advance.

Rate is expressed as—*percent.* or as a certain number cents for each hundred dollars.

Only actual loss can be recovered, so frequently only a part of the money mentioned in the policy is paid, and the insurance company may repair or pay the value of the damage.

Example 1. Insured a house worth $5000 for $3000 at $1\frac{1}{4}\%$. What was the premium? Face Policy × Rate = Premium. $\$3000 \times \frac{1\frac{1}{4}}{100} =$?

$$\overset{15}{\cancel{\$3000}} \times \frac{1}{\cancel{400}\,2} = \frac{\$15}{2} = \$7\tfrac{1}{2}. \quad \text{Answer.}$$

If totally destroyed what was the loss to the company and to the owners?

Owner's Loss—House valued at $5000—Received from Company $3000, gives loss of $2000; plus cost of Policy $7\frac{1}{2} = \$2007\frac{1}{2}$. Entire loss.

Company's Loss—Company gives $3000, but received $7\frac{1}{2}$ on policy, so loses $2002\frac{1}{2}$.

The number of years that the place is insured for does not enter into the example unless the annual rate is specified; otherwise the rate given is that which is charged on three or five year policies.

2. Premium paid for insuring $\frac{2}{3}$ of the value of a house for 3 years at $.70 was $28. What was the value of the house?

Face Policy $\times$ Rate = Premium. $\quad ? \times \frac{.70}{100} = \$28.$

$$\$28 \div \frac{.70}{100} = 28 \times \frac{100}{.70} = \$4000. \quad \text{Answer.}$$

Face of policy $4000 was only $\frac{2}{3}$ value of house, hence $\frac{1}{3}$ must have been one-half of that and the whole 3 thirds, 3 times as much or $\$4000 \div \frac{2}{3} = \overset{2000}{\cancel{4000}} \times \frac{3}{2} = \$6000.$

3. Insured property for $15,000 for 3 years, paying $92.50, which included the cost of the policy, $2.50. What was the rate? Solution: $92.50 — $2.50 = $90 Premium.

Face $\times \frac{R}{100}$ = Premium. $\quad 15{,}000 \times ? = \$90. \quad \frac{90}{15000} = .006$ or $\frac{6}{10}\% = \frac{3}{5}\%$.

4. A house worth $8000 was insured for $\frac{3}{4}$ of its value at $2\frac{1}{4}\%$, so as to include premium if destroyed; for what amount was it insured?

Rule—To find what amount must be insured to cover the premium in case of loss, divide the valuation of the property by 1 minus the rate.

Value $\div$ (1 — R) = amount covering premium. ($\frac{3}{4}$ of 8000 $\div$ (1 — $2\frac{1}{4}\%$) = ?

The sum insured = 100%. $2\frac{1}{4}\%$ is paid out so only $97\frac{3}{4}\%$ is received.

Insured for ($\frac{3}{4}$ of 8000) or $6000, which represents only $97\frac{3}{4}\%$, or $\frac{391}{400}$. If the 6000 is 391 four hundreds; one, will be as much as 391 is contained in 6000 or $15\frac{135}{391}$, and four hundred will be 400 times that or $6138.107 the amount to be insured for, so as to cover the premium and 6000.

Life Insurance

This is an agreement whereby an Insurance Company will pay a certain sum of money to a person when he reaches a certain age or in case of death to his heirs, or to some persons named in the policy, as beneficiaries. There are different kinds of policies.

a. **Ordinary Life**—the face of the policy is paid only upon the death of the person insured, who annually pays a premium to the Company for that time.

b. **Limited Life**—the face of policy is paid only upon death of the person insured who pays the premium for a certain stated number of years, usually 20, after which time nothing is paid. This is known as a "paid-up" policy.

c. **Endowment**—the face of the policy is paid to the person insured after a specified time—15, 20 or 30 years—or to his heirs in case of his death. The premiums are paid annually for the stated time.

The rates on the last two kinds of policies are higher than on the first kind.

Terms—Face of policy corresponds to **base.** The **rate** is usually so many dollars and cents per $1000—the older a person is upon becoming insured, the higher rate he will have to pay; the Company being supposed to be taking a greater risk. Insurance companies use tables giving the amount of premium paid on $1000 for each age. The **premium** corresponds to the **percentage.** It is paid generally at the beginning of the year. If the company has made any profits a dividend is sometimes declared, and this is credited to the person at the end of the year. Premiums are also frequently paid quarterly or semi-annually.

Rule—Face Policy $\times$ Rate = Premium. The rate is by the $1000, so to find the rate for $1 divide by 1000. Multiply the number of thousands in the face by the premium for $1 and it will give the premium.

Example 1. What must I pay annually for a policy of $10,000, the premium being $31.50?

Care must be taken in the use of the word premium, as in examples same as above the word "rate" is frequently omitted, the above meaning that the rate is $31.50 per $1000?

Face $\times \frac{R}{100}$ = Premium. $\quad \$10{,}000 \times \frac{\$31.50}{1000} = ? \quad \overset{10}{\cancel{10{,}000}} \times \frac{31.50}{\cancel{1000}} = \$315.$ Ans.
$\qquad\qquad\qquad

Trade or Commercial Discount

List Price—corresponds to the **base** and is the price quoted by dealers in their catalogues, etc., but it is only considered the **base** in reckoning the *first discount.*

Discounts—correspond to the **rate** and are indicated either as percents or as fractions.

The list price is usually the highest value, and this may vary with the charges in the market, therefore a discount or certain %, according to the market value, is taken off the list price. Sometimes additional discounts are allowed for cash payment or payment within a certain time. If successive discounts are allowed the first is taken from the list price and the second from the remainder, and so on.

Net Price—is the price after all deductions have been made.

Example 1. Find the net price of an automobile costing $3800 with discounts of 25% and 10%.

Discount

(a.) List Price $-\left(\text{L. P.} \times \frac{\text{R}}{100}\right) = \text{Net Price.}$ $\quad \$3800 - \left(\$38\cancel{00} \times \frac{25}{1\cancel{00}}\right) = ?$

$\$3800 - \$950 = \$2850$ Ans.

(b.) L. P. $-\left(\text{L. P.} \times \frac{\text{R}}{100}\right) = \text{Net Price.}$ $\quad \$2850 - \left(\$28\cancel{50} \times \frac{1\cancel{0}}{1\cancel{00}}\right) = ?$

$\$2850 - \$285 = \$2565$ Answer.

A shorter method is—L. P. $\times$ (100% — R%) $\times$ (100% — R%) = Net Price.

The price is represented by 100%, but not all of this is paid, a discount of 25% being allowed, hence (100% — 25%) or only 75% was paid. Then that became the whole price to be paid. Price represented by 100%, but again 10% was allowed, so only 90% was paid, which gives us $\overset{95}{\cancel{3800}} \times \frac{\overset{3}{\cancel{75}}}{1\cancel{00}} \times \frac{9}{1\cancel{00}} = \$2565$ Net Price. Ans.

2. Goods were marked down 25% from list price and then 4% was given for cash. What was the entire discount by percent?

Goods were worth 100% — discount 25% $\therefore$ goods cost 75% or $\frac{3}{4}$, which being the price to be paid was equivalent to 100%. From this 4% discount was allowed, making only $\frac{96}{100}$ of the $\frac{3}{4}$ to be paid; hence $\frac{\overset{6}{\cancel{96}}}{1\cancel{00}} \times \frac{3}{\cancel{4}} = \frac{18}{25}$ Net Price.

List Price — Net Price = Discount. $\quad \frac{25}{25} - \frac{18}{25} = \frac{7}{25} = .28$ or 28%. Answer.

3. What one discount is equal to two successive discounts of 25% and 10%?

The price is 100% — discount 25% $\therefore$ price to be paid 75%.

Then from the price 100% a discount of 10% is allowed, making 90% to be paid, so

$\frac{\overset{3}{\cancel{75}}}{1\cancel{00}} \times \frac{9}{10} = \frac{27}{40} = .675$ and 100% — $67\frac{1}{2}\% = 32\frac{1}{2}\%$. Answer.

Interest

Is the money paid for the use of money for a certain time.

Principal is the money on which the interest is charged and corresponds to the **base.**

Rate is the percent paid for the use of money for one year. The **legal rate** is the one fixed by law. In most States it is 6%. If a higher rate than the legal rate is charged it is known as **usury.** If no rate is mentioned the legal rate is understood to be meant.

Time is the period during which the principal draws interest. In ordinary interest we count 12 months to a year; 30 days to a month; 360 days to a year.

Amount is the sum of the principal and interest.

Having the product and two of the factors, the missing one is found by dividing the given product by the product of the two factors.

Interest Tables are used by banks and other business houses to compute interest.

Use of Tables—Find the desired amount at the top of the table and the time at the left hand side. The number in line with these two would be the interest for the time required.

Interest Table—Six Per Cent.

	$1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12	13.	14.	15.	16.	17	18.	19.	20.
1 year	.06	.12	.18	.24	.30	.36	.42	.48	.54	.60	.66	.72	.78	.84	.90	.96	1.02	1.08	1.14	1.20
1 month	.01	.01	.02	.02	.03	.03	.04	.04	.05	.05	.06	.06	.07	.07	.08	.08	.09	.09	.10	.10
2 "	.02	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
3 "	.02	.03	.05	.06	.08	.09	.11	.12	.14	.15	.17	.18	.20	.21	.23	.24	.26	.27	.29	.30
4 "	.02	.04	.06	.08	.10	.12	.14	.16	.18	.20	.22	.24	.26	.28	.30	.32	.34	.36	.38	.40
5 "	.03	.05	.08	.10	.13	.15	.18	.20	.23	.25	.28	.30	.33	.35	.38	.40	.43	.45	.48	.50
6 "	.03	.06	.09	.12	.15	.18	.21	.24	.27	.30	.33	.36	.39	.42	.45	.48	.51	.54	.57	.60
1 day	....	....	....	....	....	....	....	....	....	....	....	....	..	....	....	....	....	....	....	....
2 "	....	....	....	....	....	....	....	....	....	....	....	....	....	....	.01	.01	.01	.01	.01	.01
3 "	....	....	....	....	....	....	....	....	...	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
4 "	....	....	....	....	....	....	....	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
5 "	....	....	....	....	....	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.02	.02	.02
6 "	....	....	...	....	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.02	.02	.02	.02	.02	.02
7 "	....	....	....	....	.01	.01	.01	.01	.01	.01	.01	.01	.02	.02	.02	.02	.02	.02	.02	.02
8 "	....	...	....	.01	.01	.01	.01	.01	.01	.01	.01	.02	.02	.02	.02	.02	.02	.02	.03	.03
9 "	....	....	....	.01	.01	.01	.01	.01	.01	.02	.02	.02	.02	.02	.02	.02	.03	.03	.03	.03
10 "	....	....	.01	.01	.01	.01	.01	.01	.02	.02	02	.02	.02	.02	.03	.03	.03	.03	.03	.03

Example—To find interest for $135 for 5 months at 6%.

Interest for $100—Int. for $10 for 5 months, 25c. $\times$ 10 = $2.50
" " 20 for 5 months..... .50
" " 15 for 5 months..... .38
Ans. $3.38

What is the interest on $2500 for 2 years 7 months 12 days at 6%.

Formula—Principal $\times \frac{\text{rate}}{100} \times \frac{\text{time}}{360}$ = Interest.

The time is expressed in years or as part of a year—2 years = 720 days plus 7 months or 210 days plus 12 days, equals 942 days.

$$\overset{5}{\cancel{2500}} \times \frac{6}{\cancel{100}} \times \frac{\overset{157}{\cancel{942}}}{\underset{\cancel{60}}{\cancel{360}}} = \frac{785}{2} = \$392\tfrac{1}{2} \text{ Interest.} \quad \text{Answer.}$$

2. What principal will in 2 years 7 months 12 days, at 6% yield $392.50 interest?

$$P \times \frac{R}{100} \times \frac{\text{Time}}{360} = \text{Interest.} \qquad ? \times \frac{6}{100} \times \frac{\overset{157}{\cancel{942}}}{\underset{10}{\cancel{360}}} = \$392\tfrac{1}{2}$$

$$\frac{157}{1000} = \$392\tfrac{1}{2} \qquad \$392\tfrac{1}{2} \div \frac{157}{1000} = \frac{\overset{5}{\cancel{785}}}{2} \times \frac{\overset{500}{\cancel{1000}}}{\cancel{157}} = \$2500 \quad \text{Answer.}$$

3. In what time will $2500 at 6% yield $392.50?

$$P \times \frac{R}{100} \times \frac{\text{Time}}{360} = \text{Interest.} \qquad \$2500 \times \frac{6}{100} \times ? = \$392\tfrac{1}{2}$$

$$\$150 \times ? = \$392\tfrac{1}{2} \qquad \$392\tfrac{1}{2} \div \frac{\$150}{1} = \frac{\overset{157}{\cancel{785}}}{2} \times \frac{1}{\underset{30}{\cancel{150}}} = \frac{157}{60} = 2\frac{37}{60} \text{ Years.}$$

$\frac{37}{\cancel{60}} \times \cancel{12} = \frac{37}{5} = 7\frac{2}{5}$ Months. mo. $\frac{2}{\cancel{5}} \times \frac{6}{30} = 12$ days. Time = 2 yrs. 7 mo. and 12 days
5

4. A man borrowed $2500 for 2 yrs. 7 mo. 12 days at 6%. What amount did he have to pay at the end of that time?

$$\text{Prin.} + \left(\text{Prin.} \times \frac{\text{Rate}}{100} \times \frac{\text{Time}}{100}\right) = \text{Amount.} \qquad \$2500 + \left(\$2500 \times \frac{6}{100} \times \frac{942}{360}\right) = ?$$

Interest

From Example (A) we know that the interest was $392½ ∴ $2500 + $392½ = $2892½

5. What principal will amount to $2892½ at 6% in 2 yr. 7 mo. 12 days.

$$\text{Prin.} + \left(\text{Prin.} \times \frac{R}{100} \times \frac{\text{Time}}{360}\right) = \text{Amount.} \qquad ? + \left(? \times \frac{6}{100} \times \frac{942}{360}\right) = \$2892\tfrac{1}{2}$$

Interest

Considering the Principal as the Base or the whole, on which the interest is to be reckoned, it may be represented by $\frac{100}{100}$ or 1. $\quad 1 \times \frac{6}{100} \times \frac{157}{360} = \frac{157}{1000}$

The Principal is again the whole represented by $\frac{1000}{1000}$

$$\frac{1000}{1000} + \frac{157}{1000} = \$2892\tfrac{1}{2}. \qquad \$2892\tfrac{1}{2} \div \frac{1157}{1000} = \frac{5785}{2} \times \frac{1000}{1157} = \frac{2892500}{1157} = \$2500.$$

6. In what time will $2500 at 6% amount to $2892½?

$$\text{Prin.} + \left(\text{Prin.} \times \frac{R}{100} \times \frac{\text{Time}}{369}\right) = \text{Amt.} \qquad \$2500 + \left(\$2500 \times \frac{6}{100} \times ?\right) = \$2892\tfrac{1}{2}.$$

Amount — Principal = Interest. $\quad \$2892\tfrac{1}{2} - \$2500 = \$392\tfrac{1}{2}.$

$$\text{Prin.} \times \frac{R}{100} \times \frac{\text{Time}}{360} = \text{Interest.} \qquad \$2500 \times \frac{6}{100} \times ? = \$392\tfrac{1}{2}$$

and resolves itself into Example 3.

7. At what rate will $2000 amount to $2892½ in 2 yrs. 7 mo. 12 days?

$$P. + \left(P. \times \frac{R}{100} \times \frac{\text{Time}}{360}\right) = \text{Amount.}$$

Amount — Principal = Interest. $\quad \$2892\tfrac{1}{2} - \$2500 = \$392\tfrac{1}{2}.$

$$P \times \frac{R}{100} \times \frac{\text{Time}}{360} = \text{Int.} \qquad \$2500 \times \frac{?}{100} \times \frac{942}{360} = \$392\tfrac{1}{2},$$

and resolves itself into Example 4

Difference in Time

In examples where two dates are given—to find the time, 360 days are always reckoned to the year and 30 days to the month. The time being found by subtracting the given dates; as what is the time from July 20th to October 10th?

October 10 = 10 months — 10 days. $\quad$ July 20 = 7 months — 20 days.

We cannot subtract 20 days from 10 days, so subtracting 1 month from 10 months leaves 9 months and 40 days; 20 days from 40 days gives 20 days; 7 months from 9 months gives 2 months.

(United States Government and some Bankers in their interest for days take the actual number of days to the month and year.)

Exact Interest

Examples are worked exactly the same as for ordinary interest, with the exception that the actual number of days between dates is taken and 365 days reckoned to the year.

$$\text{Formula} — \text{Principal} \times \frac{\text{Rate}}{100} \times \frac{\text{Time (exact)}}{365} = \text{Exact Interest.}$$

Six Percent Method—(Interest)

If one dollar can earn six cents in one year, in one month it will earn one-twelfth of six cents or one-half of one cent. If it earns half a cent in one month, in two months it will earn one cent, or as much expressed in cents as there are months divided by two.

If in one month one dollar earns one-half cent or five mills, in six days, which is one-fifth of one month, it will earn one-fifth of five mills, which is one mill, and in one day it will earn one-sixth of a mill. Therefore if you divide the number of days by six and express the quotient as mills you will have what one dollar earns in the given number of days. Therefore *the six percent method may be taught as follows:*

To find the interest on one dollar for the number of years, multiply the number of years by .06 and express the product as cents.

To find the interest on one dollar for a given number of months divide the number of months by two. Express the quotient as cents. This will be the interest for the given number of months.

To find the interest for a given number of days divide the number of days by six. Express the quotient as mills. This will be the interest for the given number of days.

Add the interest for the years, months, and days together. This will be the interest on one dollar for the given time. Multiply the sum upon which the interest is to be found by the interest on one dollar for the given time, and the product will be the interest required.

If the **rate is five percent** proceed as in the six percent method, and after finding the interest at six percent subtract one-sixth of the interest from the whole amount and the remainder will be the interest at five percent. If the rate is **seven percent** add one-sixth of the interest found by the six percent method and the amount will be the interest at seven percent.

Example—Find the interest on $50.25 for 2 yrs. 7 mo. 17 days at 6%?

Interest $1 for 2 yrs. will be .06 × 2 or $.12. Interest $1 for 7 mo. will be 7 divided by 2 or $.03½. Interest $1 for 17 days will be 17 divided by 6 or $.002⅚. Interest for the given time will be $.12 + $.035 + $.002⅚ = $.157⅚. And for $50.25 will be 50.25 × $.157⅚ = $7.93 Answer.

Present Worth and True Discount

Examples in Present Worth and True Discount are practically examples in Interest in which **present worth** corresponds to the **principal.** It is the present value of a debt, or such a sum of money, which put at interest for the given time would amount to the debt.

Rate—number percent given as the value of the money.

Time—period of time showing when the debt is due.

True discount corresponds to the **interest** on the present worth for the time between its payment and when it is due.

Debt corresponds to the **amount.**

Example 1. What is the present worth of $605, payable 3 yrs. 6 mo. from now, money being worth 6%. True discount?

$$\text{Prin.} + \left(\text{Prin.} \times \frac{R}{100} \times \frac{\text{Time}}{360}\right) = \text{Amt.} \qquad ? + \left(? \times \frac{6}{100} \times \frac{1260}{360}\right) = \$605.$$

3 yrs. = 1080 days
6 mo. = 180 "
1260 "

Assuming the principal to be the whole, or $\frac{100}{100}$, we have

$$\text{(a.)}\ \frac{\cancel{100}}{100} \times \frac{\cancel{6}}{\cancel{100}} \times \frac{\overset{21}{\cancel{1260}}}{\underset{\cancel{6}}{\cancel{360}}} = \frac{21}{100} \qquad \text{(b.)}\ \frac{100}{100} + \frac{21}{100} = \$605.$$

$$\text{(c.)}\ \$605 \div \frac{121}{100} = \$605 \times \frac{100}{121} = \frac{60500}{121} = \$500$$

Amount of Debt — Present Worth = True Discount. $605 — $500 = $105. Ans.

Bank Discount

Face is the sum of money named in the note.

Rate is the certain number of percent of the face. If no percent is mentioned, *six*, or the legal rate of the State, is understood.

Term of discount is the number of days between the day the note is discounted and the day the note is due.

Days of Grace—Some States allow three extra days in which to pay a note. Where this is allowed add (3) three to the number of days in the term of discount.

Date of maturity is the day on which the note becomes due. Notes are usually drawn for 30, 60 or 90 days; or 2 or 3 months. If the day of maturity falls on Sunday or legal holiday, it becomes due the previous day in some and in other States the day after.

Bank Discount is the interest on the face of the note for the time from the day of discount to the day when due. This interest is collected in advance by the one discounting the note in consideration for having advanced the money before it was due.

Proceeds is the sum received for the note after it has been discounted. It is found by subtracting the discount from the face of the note (or the amount if the note is interest-bearing).

If the note is interest-bearing the bank discount is found not on the face of the

note, but on the full debt, the face, plus the interest that would be due on the note at maturity.

A business man will buy goods and not wish to pay for them immediately, so gives a note covering the amount. The merchant from whom the goods were bought and who received the note may need the money and will then endorse the note and take it to the bank and have it discounted, receiving not the face of the note or the sum mentioned in the note, but that amount less the discount for the time which the note has still to run before it is due.

Notes

A **note** is the written promise to pay to a person named in it, on demand or at some specified time, a designated sum of money or article mentioned in it.

A **note should contain** name of place where it is made; date when it is made; length of time for which it is made. (This may be:—On demand; easily determined time; specified time); name of person in whose favor it is made; the amount in words; the amount in figures; place where it is to be paid. If no place is mentioned then it is understood to be the place of business or residence of the maker of the note; it may contain the words "for value received"; the signature of the one making the note. If the note is to draw interest it should contain the words "*with interest.*"

The **maker** is the person signing the note. The **payee** is the person in whose favor it is made. The **holder** is the person who holds the note.

A **negotiable note** is one that can be bought or sold. It should contain the words "*bearer*" or "*order of.*"

A **non-negotiable note** is one that cannot be bought or sold. It does not contain the words "*bearer*" or "*order of.*"

Protest is the formal notice sent to the maker and endorser that the payment of a check or note has been demanded and refused. It is made for the purpose of holding endorsers responsible. It is done by a notary on the day the note or check is due. Mailing this notice to the persons whose names appear on the check or note is considered notice of protest.

A note that is payable at a bank *must be presented for payment* **at the time** *it becomes due*, else payment will be refused, although the maker of the note may have the money on deposit in the bank.

Formulas:

$$\text{Face} \times \frac{R}{100} \times \frac{\text{Time days}}{360}\ (\textit{Term Discount}) = \text{Bank Discount.} \qquad \text{Face} - \text{Bank Discount} = \text{Proceeds}$$

Simple interest is being found, so use 360 as a denominator, though the numerator represents the actual number of days.

For an interest-bearing note:

(a.) $\text{Face} \times \frac{R}{100} \times \frac{\text{Time to run}}{360} = \text{Interest.}$ (b.) $\text{Face} + \text{Interest} = \text{Debt.}$

(c.) $\text{Debt} \times \frac{R}{100} \times \frac{\text{Term Discount}}{360} = \text{Bank Discount.}$

(d.) Debt — Bank Discount = Proceeds.

$300 STRATFORD, N. Y., June 5, 1907.

Ninety days after date, I promise to pay Charles Smith, or order, Three Hundred Dollars, at the Brooklyn Trust Company, for value received. JOHN BROWN.

Discounted July 15, 1907, at 6%. Find Bank Discount and Proceeds on the above.

Date—June 5, 1907

90 days | Discounted July 15, at 6%

50 days (Term Discount)

Date Maturity—Sept. 3

Date of note—June 5, 1907 + 90 days = Date Maturity, or September 3, 1907.

Date of Discount— July 15, 1907, to Date of Maturity—September 3, 1907, equals Term Discount = 50 days.

	Days.
June	25 left
July	31
Aug.	31
Sept.	3 (Date Maturity)
	90 days

$$\text{Face} \times \frac{\text{Rate}}{100} \times \frac{\text{Term of Discount}}{360} = \text{Bank Discount.}$$

July 16 left
Aug. 31
Sept. 3
50 days (Term Discount)

$$\$3\not{0}\not{0} \times \frac{\overset{5}{\not{6}}}{1\not{0}\not{0}_{2}} \times \frac{5\not{0}}{3\not{6}\not{0}_{\not{6}\not{0}}} = \frac{5}{2} = \$2.50$$

Face — Bank Discount = Proceeds. $300 — $2.50 = $297.50

2. A note for $300 due in 90 days was discounted immediately at 6%. What was the discount and what was the proceeds?

90 days | Date: Discount | 90 Term Discount | Maturity

$$\text{Face} \times \frac{\text{Rate}}{100} \times \frac{\text{Term of Dis.}}{360} = \text{Bank Discount}$$

$$\overset{3}{3\not{0}\not{0}} \times \frac{\not{6}}{1\not{0}\not{0}} \times \frac{9\not{0}}{3\not{6}\not{0}_{\not{6}\not{0}}} = \frac{9}{2} = \$4.50$$

Face — Bank Discount = Proceeds. $300 — $4.50 = $295.50

3. Find the face. Wishing to borrow $295.50 from a bank for 90 days, for what must I give my note, discount 6%?

$$\text{Face} \times \frac{\text{Rate}}{100} \times \frac{\text{Term Discount}}{360} = \text{B. D.}$$

Face — Bank Discount = Proceeds; or

$$\text{Face} - \left(\text{Face} \times \frac{\text{Rate}}{100} \times \frac{\text{Term of Discount}}{360}\right) = \text{Proceeds}$$

$$? - \left(? \times \frac{6}{100} \times \frac{90}{360}\right) = \$295.50$$

Assuming the face to be the whole amount or 1, it may be expressed by $\frac{100}{100}$ or any fraction equivalent to 1.

(a.) $\frac{1\not{0}\not{0}}{1\not{0}\not{0}} \times \frac{\overset{3}{\not{6}}}{100} \times \frac{9\not{0}}{3\not{6}\not{0}_{2}} = \frac{3}{200}$ (b.) $\frac{200}{200} - \frac{3}{200} = \$295.50$

$$\$295.50 \div \frac{197}{200} = \$295.50 \times \frac{200}{197} = \frac{59100}{197} = \$300$$

4. A man buys a 90-day note for $4.50 less than its face; what was the face the discount being 6%?

$$\text{Face} \times \frac{\text{Rate}}{100} \times \frac{\text{Term Discount}}{360} = \text{B. D.}$$

(a.) $? \times \frac{\overset{3}{\not{6}}}{100} \times \frac{9\not{0}}{3\not{6}\not{0}_{2}} = \$4.50$

(b.) $? \times \frac{3}{200} = \$4\frac{1}{2}$ (c.) $\$4\frac{1}{2} \div \frac{3}{200} = \frac{\overset{3}{\not{9}}}{\not{2}} \times \frac{\overset{100}{2\not{0}\not{0}}}{\not{3}} = \$300$ Face.

5. A 90-day note for $300 was discounted at 6%, 50 days before it was due. What was the bank discount?

$$\text{Face} \times \frac{\text{Rate}}{100} \times \frac{\text{Term Discount}}{360} = \text{Bank Discount}$$

$$\$3\not{0}\not{0} \times \frac{\overset{5}{\not{6}}}{1\not{0}\not{0}_{2}} \times \frac{5\not{0}}{3\not{6}\not{0}_{\not{6}\not{0}}} = \$2\tfrac{1}{2} \text{ Ans.}$$

$300 STRATFORD, New York, June 5, 1907.

Three months after date I promise to pay Charles Smith or order, Three Hundred Dollars with interest at 5% at the Brooklyn Trust Company for value received.

JOHN BROWN.

Discounted July 15, 1907, at 6%.

3 months | Date June 5 | Discounted July 15 at 6% | 52 days (Term Discount) | Maturity September 5

$$\text{Face} \times \frac{\text{Rate}}{100} \times \frac{\text{Time}}{360} = \text{Interest.}$$

$$\overset{3}{\$3\not{0}\not{0}} \times \frac{5}{1\not{0}\not{0}} \times \frac{9\not{0}}{3\not{6}\not{0}_{4}} = \frac{15}{4} = \$3.75$$

July 16 (left)
Aug. 31
Sept. 5
52

Face + Interest = Full Debt.
300 + 3.75 = $303.75

$$\text{Debt} \times \frac{R}{100} \times \frac{\text{Term of Discount}}{360} = \text{Bank Discount.} \qquad 303.75 \times \frac{6}{100} \times \frac{13}{360} = \frac{10.53}{4} = \$2.63$$

$$\text{Debt} - \text{Bank Discount} = \text{Proceeds.} \qquad \$303.75 - \$2.63 = \$301.12$$

6. Find the proceeds of note for \$148.50. Dated October 16. Due December 15. Interest 6%. Discounted 14 days before due. Rate of discount 6%.

$$F \times R \times T = \text{Int.} \qquad \$148.50 \times \frac{6}{100} \times \frac{60}{360} = 1.485$$

	October 16	Date
60	December 1	Discounted
	14 days	(Term Dis.)
	December 15	Maturity

$$\text{Face} + \text{Int.} = \text{Debt.} \qquad \$148.50 + 1.49 = \$149.99$$

$$\text{Debt} \times R \times T = \text{B. D.} \qquad \$149.99 \times \frac{6}{100} \times \frac{14}{360} = \frac{10.4993}{30} = \$.349$$

Oct.	15
Nov.	30
Dec.	15
	60 days

$$\text{Debt} - \text{Bank Discount} = \text{Proceeds.} \qquad \$149.99 - .35 = \$149.64$$

7. Proceeds is \$149.64 of a 6% note for 60 days, which is discounted at 6%, 14 days before maturity; what was the face?

$$\text{Debt} - (\underset{\text{Discount}}{\text{Debt}} \times R \times T) = \text{Proceeds.} \qquad ? - \left(? \times \frac{6}{100} \times \frac{14}{360} \right) = \$149.64$$

(a.) $\frac{100}{100} \times \frac{6}{100} \times \frac{14}{360} = \frac{7}{3000}$ (b.) $\frac{3000}{3000} - \frac{7}{3000} = \frac{2993}{3000} = \$149.64$

(c.) $\$149.64 \times \frac{2993}{3000} = 149.64 \times \frac{3000}{2993} = \frac{448920}{2993} = \$149.99$

$$F + (F \times R \times T) = \text{Debt.} \qquad ? + \left(? \times \frac{6}{100} \times \frac{60}{360} \right) = \$149.99$$

(a.) $\frac{100}{100} \times \frac{6}{100} \times \frac{60}{360} = \frac{1}{100}$ (b.) $\frac{100}{100} + \frac{1}{100} = \frac{101}{100} = \$149.99$

(c.) $\$149.99 \div \frac{101}{100} = \$149.99 \times \frac{100}{101} = \frac{14999}{101} = \$148.50$

Exchange

This is payment of money by means of money-orders, checks, or drafts when the person to whom the money is sent lives in another place. If the amount is large a draft is generally used. It is the same as a check, but is made by the cashier of one bank and drawn on another bank.

A bank usually charges a premium on the face of a draft.

Maker—person who signs the draft, etc. **Drawee**—person asked to pay the draft. **Payee**—person to whom the money is paid. **Acceptance**—promise of the person requested to pay that he will pay when the draft is presented to him. He writes "*accepted*" with his name and date across the draft.

Drafts sell according to the trade between the two places; as, either at

Par—When a draft sells at its face and there is no rate, or **Premium**—When a draft sells for more than the face and the rate is added to *par*, or **Discount**—When a draft sells for less than its face and the rate is subtracted from *par*.

Terms used in Exchange: Face—which corresponds to the **base. Rate**—which is a certain percent of the face or so many cents per \$1000. **Premium or Discount**—which corresponds to the *percentage*. **Cost of Draft**—is the **amount** if sold at a premium, or the **difference** if sold at a discount.

Example 1. What would a draft for $300 cost at .1% premium?

$$\text{Face} \times \frac{R}{100} = \text{Premium.} \qquad \text{Face} + \left(\text{Face} \times \frac{\text{Rate}}{100}\right) = \text{Cost of Draft.}$$

$$300 + \left(\overset{3}{\cancel{300}} \times \frac{.1}{\cancel{100}}\right) = ? \qquad \$300 + .3 = \$300.30 \quad \text{Answer.}$$

2. Find cost of a draft for $3000 at 75 cents discount per $1000?

$$\text{Face} - (\text{Face} \times \text{Rate}) = \text{Cost Draft.} \qquad \$3000 - \left(\overset{3}{\cancel{3000}} \times \frac{.75}{\cancel{1000}}\right) = \$3000 - \$2.25 = \$2999.75$$

$300 Brooklyn, N. Y., June 5, 1910.

At sight pay to the People's Trust Company or order Three hundred Dollars. Value received, and charge to the account of

To John Smith. Ed. Brown.

Mr. Smith owes Mr. Brown $300; the latter being in need of the money draws on Mr. Smith by means of a draft similar to the above. Mr. Brown takes it to the People's Trust Company, who cash the draft and he receives the money. The bank then sends the draft to a Boston Bank to collect from Mr. Smith. If the money is obtained it is sent to the Brooklyn Bank. If payment is refused for any reason, the draft is sent back to the Brooklyn Bank and they send notice to Mr. Brown that he must "*take up the draft*," which means that he must give back to the bank the money they gave him on it. The bank charges for collection and its trouble, this charge being deducted from the face of the draft.

Corporation

Several persons united into one body for business purposes, with rights and liabilities the same as an individual.

Shares—Stockholders—The money to carry on the business is given by the members of the corporation. It is divided into *shares*, usually a $100 each, and any person holding a share is known as a *stockholder*. Every stockholder should receive a certificate duly stamped with a government stamp showing how many shares he owns.

Directors—The stockholders elect from among their number, certain ones to act as directors in conducting the business, and these in turn elect the officers of the company.

Dividends—When all expenses are paid and a sufficient amount is laid aside for current expenses the remainder is considered profit, and is divided among the stockholders according to the amount of stock held; this money is known as the *dividend*.

A company with $50,000 capital makes a profit of $2000 and gives a dividend of 4%. Therefore a person holding $10,000 worth of stock would receive $400 as his dividend.

$$\$10{,}000 \times \frac{4}{100} = \$400$$

Partnership

Two or more people associated together in business. The amount gained or lost is divided among the partners in proportion to the amount of stock they hold.

Example—A and B form a partnership. A puts in $5000 and B $3000. They gained $800; what was each man's share of the profits?

$5000 + $3000 = $8000 Entire Capital $800 = Entire Gain

$\frac{5000}{8000} = \frac{5}{8}$ A's share of Capital $\frac{5}{\cancel{8}} \times \frac{100}{\cancel{800}} = \$500$ A's gain

$\frac{3000}{8000} = \frac{3}{8}$ B's share of Capital $\frac{3}{\cancel{8}} \times \frac{100}{\cancel{800}} = \$300$ B's gain

} Answer

Stocks and Bonds

A share of stock is usually considered to represent $100. If the stock sells at that price it is said to be at *par*. If the investment is a good one people will be anxious to secure that stock and will pay more than $100 a share; it is then said to be *above par*; but on the contrary, if the investment does not bring in good returns people will not care to have it and the stock is sold for less than $100, and so is said to be *below par*.

Stocks—Preferred and Common

Preferred Stock—that which has a fixed rate of dividend that must be paid before any other dividends can be paid—usually 5 or 7%.

Common Stock—that on which a share of the net earnings are paid after the dividends on *preferred stock* have been paid. If the earnings are large this may be more valuable than preferred stock, often giving 10%, but it is not to be counted on, and therefore is not as safe as preferred stock.

Buying or selling stock is usually done by a broker, who charges $\frac{1}{8}\%$ of par value for his services, both from the purchaser and seller.

Newspapers each day give the *stock quotation*, that is, the value of the various stocks, so that people may keep track of their investments to know whether they are selling at par, above, or below, and how much.

So if a certain railroad stock is quoted as $113\frac{1}{4}$ it means that a \$100 worth of stock would cost \$113.25 and a person wishing to buy at that figure would have to pay in addition the brokerage fee $\frac{1}{8}\%$ on \$100 or $\$.12\frac{1}{2}$, making a total of $\$113.37\frac{1}{2}$ for each share; the person selling would however receive only $\$113.12\frac{1}{2}$, as he would also have to give the $\frac{1}{8}\%$ of \$100 to the broker.

Fractions are always $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{8}$. A part of a share cannot be bought.

Example—A man buys 50 shares of stock at $137\frac{1}{2}$ and sells at $139\frac{1}{8}$, how much does he gain?

Cost + Brokerage $137.25 + .12\frac{1}{2} = 137.37\frac{1}{2}$

Selling Price — Brokerage. $139.12\frac{1}{2} - .12\frac{1}{2} = \$139$

$139 - 137.37\frac{1}{2} = 1.62\frac{1}{2}$ Gain on 1 share.

$$\$1.62\tfrac{1}{2} \times 50 = \frac{13}{8} \times \frac{\overset{25}{\cancel{50}}}{1} = \frac{325}{4} = \$81\tfrac{1}{4} \text{ or } \$81.25 \quad \text{Total Gain. Answer.}$$

Bond is a written promise to pay a specified amount at a given time. It must be properly executed and bear the legal seal. If the bond and interest is not paid when due the holder may sell the property to cover his loss. He holds a mortgage on the property which is issued at the same time as the bond in order to make his claim secure.

Stockholder owns property but a ***bondholder*** is one to whom money is due.

Incomes or dividends for stocks or bonds are always reckoned on par value,

Example—What is the dividend on 30 shares of stock at 6%?

Par value 1 share = 100 Par value 30 shares = 3000 $3000 \times 5\% = \$150$ Ans.

Example—What is the rate of income on 5% bonds costing $112\frac{1}{8}$?

Cost + Brokerage. $\$112\frac{1}{8} + \$\frac{1}{8} = \$112.25$ paid for the \$100 bond.

Dividend is 5% on par value or $100 \times 5\% = \$5$

$$112.25 \times ? = \$5 \qquad \$5 \div \$112\tfrac{1}{4} = 5 \times \frac{4}{449} = \frac{20}{449} = .04 \text{ or } 4\% \quad \text{Answer.}$$

Compound Interest

The principal (in a Savings Bank) yields interest which is given at certain times—every 3, 6 or 12 months. If the interest is not collected, but allowed to remain, it is added to the principal and then draws interest, this being known as *Compound Interest.*

If not stated otherwise, the rate, when given, is usually that for a year, so if interest is paid quarterly every three months, take $\frac{1}{4}$ of the rate, and, if semi-annually, take $\frac{1}{2}$ of the rate.

To find the Compound Interest, subtract the given principal from the last amount.

Example 1—If \$500 is deposited in a savings bank giving 4% interest for $1\frac{1}{2}$ years, interest compounded semi-annually, what would it amount to at the end of that time? What is the interest?

Interest 4% annually makes 2% semi-annually.

\$500	Principal
× .02	
10.00	Interest at the end of first 6 months.
500.	
510.	Amount " " " " " " "
.02	
10.20	Interest at the end of 2d-6 months.
510.	
520.20	Amount " " " " " "
.02	
10.4040	
520.	
\$530.404	Amount at the end of the 3rd-6 months. *Answer.*

\$530.40 — 500 = \$30.40 Compound Interest. Answer.

Partial Payments

The maker of a note may not be able to settle the whole debt at one time and so pays in parts or makes partial payments at different times. These are noted on the back of the note with the date and the amount.

These part payments are used first to settle any interest due, and then, if there is anything left, to reduce the principal, the idea being that neither interest nor payment shall draw interest. This was so decided by the Supreme Court of the United States, and most states use this method.

In finding the time between the dates, subtract years, months, and days.

Example 1—A note for $500 at 5% is dated June 5, 1907. The following payments were made: July 1, 1907, $10; January 7, 1908, $10; April 10, 1908, $75. How much was due June 5, 1908?

Principal	$500.	
Time—June 5, 1907, to July 1, 1907 = 26 days.		
Interest for above time on $500	1.81	
Total	$501.81	
First payment	10	
New principal	$491.81	
Time from July 1, 1907, to January 7, 1908 = 190 days		
Interest for above time	12.98	
Second payment, $10, is less than the interest due		
Time from January 7, 1908, to April 10, 1908 = 94 days.		
Interest for the above on 491.81	6.42	
Total	$511.21	
Third payment, $75, added to 2nd, $10	85.00	
New principal	$426.21	
Time from April 10, 1908, to June 5, 1908 = 56 days.		
Interest for above time on new principal	22.21	
New principal or amount due June 5, 1908	$448.41	Answer.

Involution—Squaring Numbers

Squaring a number is multiplying it by itself. The result is a **Square.** Squaring is indicated by a small 2 written to the right and above the number; as $2^2 = 4$

The square of a number of one figure contains one or two figures, the square of a number of two figures contains three or four figures. The square of any number contains twice as many figures or twice as many figures less one, as the number itself.

Any number of two figures or more, may be resolved into tens and units.

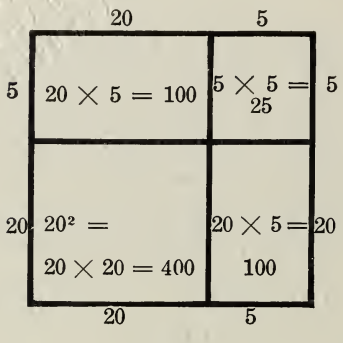
$$(t + u)^2 = t + u$$
$$t + u$$
$$tu + u^2$$
$$t^2 + tu$$
$$t^2 + 2tu + u^2$$

$$25^2 = (2 \text{ tens} + 5 \text{ units})^2$$
$$2t + 5u$$
$$2t + 5u$$
$$10tu + 25u^2$$
$$4t^2 + 10tu$$
$$4t^2 + 20tu + 25u^2$$

From the above we derive the following—square of the tens, plus twice the product of the tens by the units, plus the square of the units, equals the square of a number that has been resolved into tens and units.

$3^2 = 3 \times 3$ or 9

Diagram



$25^2 = 25 \times 25 = 625$
$25 = 20 + 5$ or 2 tens + 5 units
$_____ = 5$ units
$_____ = 2$ tens or 20
$_____ = 2$ tens + 5 units or $20 + 5$

square of 25 =
$20 \times 20 = 400$ or t^2
$20 \times 5 = 100$ or $t \times u$
$20 \times 5 = 100$ or $t \times u$
$5 \times 5 = 25$ or u^2

625 or $(t^2 + 2\ (t \times u) + u^2)$

Square Root

Sqnare Root of a number is finding one of the two equal factors. The **sign** of square root is the radical; as $\sqrt{49}$ or $\sqrt[2]{49}$ which would be 7^2, for $7 \times 7 = 49$.

$$625 = \begin{cases} t^2 + (2t + u)\ u \\ t^2 + 2\ (t \times u) + u^2 \end{cases}$$

$t^2 + 2\ (t \times u) + u^2$

6 2 5 | t u
2 5

$t^2 = 4$
$2t \times u = \quad | 225$
$40 \times 5 = 200 |$
$u^2 = 5^2 = 25 | 225$

$t^2 + (2t + u)\ u$

6 2 5 | t u
2 5

$t^2 = 4$
$2t + u = 40 + 5 = 45 | 225$
$\times u = \times 5$ or $| 225$

Rule for Extracting the Square Root

Separate the number representing a square into periods of two figures each, beginning at the decimal point, and the number of periods will be the same as the number of figures in the square root. The left hand period sometimes only contains 1 figure.

Beginning at the decimal and having separated the square into periods of 2 figures each, find the greatest square contained in the left hand period and subtract it, bringing down the next period. Place in the quotient the square root of the square just used.

Multiply the answer so far obtained by 2; using this product as a trial divisor see how many times it is contained in the new dividend. Place this figure in the answer beside the last one found. Add this last figure to the trial divisor by placing it at the right of the figure already in the divisor and multiply the whole divisor by the last figure.

Subtract the result and bring down the next period, continue as before till the end of the example.

If a zero occurs in the root add a cipher to the trial divisor, bring down the next period and continue as before.

Square Root of Decimals—point off periods of 2 places each as in whole numbers, beginning at the decimal point, moving to the right of the decimal point for the decimal part, and to the left for the whole numbers. If more decimal places are desired annex 2 ciphers for each period, and continue as for whole numbers.

Square Root of Fraction—find the square root of each term, numerator and denominator. If they are not perfect squares it is better to reduce the fraction to a decimal before extracting the square root.

Application of Square Root

1. To find the side of a square.

Rule—Given the area of a square to find one side, extract the square root.

Example—A man has a farm containing 10 acres. What is its length in rods?

?

10 A.

or

1600 sq. rds.

In linear measure for the length of a line there are no acres, so we must reduce 10 acres to square rods by multiplying by 160, which gives us 1600 square rods. Then extracting the square root we will have the number of rods on a side.

$\sqrt{1600}$ = 40 rds.

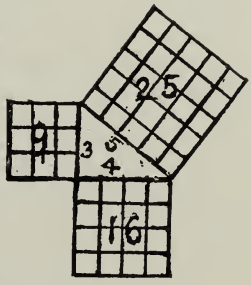
16

00

00

II—Right-Angled Triangles

Base (B) side on which triangle rests. **Perpendicular** (P) side with which the base forms a right-angle. **Hypothenuse** (H) side opposite the right-angle.



Principles

1. **Square of the hypothenuse** equals the sum of the square of the other two sides.
2. **Square of the base** equals the square of the hypothenuse minus the square of the perpendicular or altitude.
3. **Square of the altitude** equals the square of the hypothenuse minus the square of the base.

Formulas (1) $\sqrt{B^2 + P^2} = H$ (2) $\sqrt{H^2 - B^2} = P$ (3) $\sqrt{H^2 - P^2} = B$

Example 1. In a triangle the base is 12, the perpendicular 16; what is the hypothenuse?

(a.) $\sqrt{B^2 + P^2} = H$ (b.) $\sqrt{12^2 + 16^2} = ?$

(c.) $\sqrt{144 + 256} = ?$ (d.) $\sqrt{400} = 20$ Answer.

2. A string 20 feet reaches from the top of a pole to a point on the ground 5 ft. from the foot of the pole; what is the height of the pole?

(a.) $\sqrt{H^2 - B^2} = P$ (b.) $\sqrt{20^2 - 5^2} = ?$

(c.) $\sqrt{400 - 25} = ?$ (d.) $\sqrt{375} = 19.3 +$ Answer.

3. A rope 40 ft. long reaches from top of a tree 25 ft. high to the bottom of a fence opposite the tree; how far is it from the tree to the fence?

(a.) $\sqrt{H^2 - P^2} = B$ (b.) $\sqrt{40^2 - 25^2} = ?$

(c.) $\sqrt{1600 - 625} = ?$ (d.) $\sqrt{975} = 31.2 +$ feet. Answer.

MENSURATION

This deals with the measuring of *lines*, *surfaces* or *solids*.

A. **Surface**—anything which has length and breadth, but not thickness.

a. **Plane Surface**—surface bounded by straight lines.

b. **Curved Surface**—surface bounded by curved lines.

B. **Plane Figure** is surface bounded by either straight or curved lines.

C. **Polygon**—plane figure bounded by three or more straight lines; as, Triangle, Quadrilateral, Pentagon, etc. It takes its name from the number of sides it has.

Base is the side on which the polygon rests. **Altitude**—the perpendicular distance between its base and opposite side or angle. **Perimeter**—the sum of all the sides or the distance around the polygon. **Diagonal**—line connecting two angles not adjoining each other. **Area**—the amount of surface included within the bounding lines and is always indicated in square measure.

See Rule 2 under Triangles.

D. **Triangle**—a polygon which is bounded by three straight lines and which has three angles.

a. **Angle** is the space near the point where two lines meet. **Right Angle**—space of 90° between two lines drawn perpendicular to each other. **Acute Angle**—space less than 90° between two lines meeting at a point. **Obtuse Angle**—space more than 90° between two lines meeting at a point.

b. **Base**—side on which the triangle rests. **Altitude**—perpendicular distance between the base and the opposite angle.

c. **Kinds of Triangle: Equilateral**—one in which the sides are all equal. **Isosceles**—one in which two sides are equal. **Scalene**—one in which all sides are unequal.

d. **To find the area of a triangle.**

Rule 1. If three sides are given and not the altitude:

a. Subtract each side separately from the half sum of all the sides.

b. Extract the square root of the product of all the remainders and the half sum.

Example—What is the area of a field whose sides measure respectively 30, 40, 50 rods?

$$\frac{30 + 40 + 50}{2} = \frac{120}{2} = 60$$

$$\sqrt{60 \times (60 - 30) \times (60 - 40) \times (60 - 50)} = ?$$

$$\sqrt{60 \times 30 \times 20 \times 10} = ? \qquad \sqrt{360000} = 600 \text{ square rods. Answer.}$$

Rule 2. Half the product of the base and altitude equals the area.

$$\frac{B \times \text{Alt.}}{2} = \text{Area of triangle.}$$

Example 1. What is the area of a triangle whose base is 26 feet and whose altitude is 16 feet.

$$\frac{B \times \text{Alt.}}{2} = \text{Area.} \qquad \frac{\overset{8}{\cancel{26}} \times \overset{}{\cancel{16}}}{\underset{}{\cancel{2}}} = 208 \text{ sq. ft. Answer.}$$

Example 2. If the area of a triangle is 208 square feet and the base is 26 feet, what is the altitude?

$$\frac{B \times \text{Alt.}}{2} = \text{Area.} \qquad \frac{\overset{13}{\cancel{26}} \times ?}{\underset{}{\cancel{2}}} = 208 \text{ sq. ft.}$$

$$13 \times ? = 208 \qquad 208 \div 13 = 16 \text{ ft.}$$

Example 3. If the area is 208 sq. ft. and the altitude is 16 ft., what is the base?

$$\frac{B \times \text{Alt.}}{2} = \text{Area.} \qquad \frac{? \times 16}{2} = 208$$

$$208 \div \frac{16}{2} = \frac{\overset{26}{\cancel{208}}}{1} \times \frac{\overset{}{\cancel{2}}}{\underset{8}{\cancel{16}}} = 26 \text{ ft. Answer.}$$

E. **Quadrilateral**—a polygon bounded by four straight lines and four angles.

1. **Parallelogram**—quadrilateral whose sides are equal and parallel.
   a. **Square**—all four sides are equal and all four angles right angles.
   b. **Rectangle**—opposite sides equal and all four angles are right angles.
   c. **Rhombus**—all four sides equal with two acute and two obtuse angles.

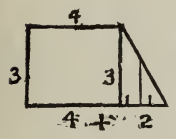
d. **Rule for Area Parallelograms**—multiply the base by the altitude expressed in like denominations. (B $\times$ Alt.) = Area.

2. **Trapezoid**—quadrilateral with only two sides parallel.

a. **Altitude** is the perpendicular distance between the parallel sides.

b. **Rule for Area**—Multiply half the product of the sum of the parallel sides by the altitude.

$$\left(\frac{\text{Sum of parallel sides}}{2}\right) \times \text{Altitude} = \text{Area.}$$



Given a trapezoid whose altitude is 3 and whose bases are respectively 6 and 4 find the area. Take 4 as the upper base and 6 as the lower base, divide the trapezoid into a rectangle, dimensions 4 by 3; and a triangle whose altitude is 3 and the base 2. Erect a perpendicular to the center point of the base of the triangle, dividing it into two parts, making each part of the base 1. Remove the small triangle just constructed and attach it base upward to the upper base of the trapezoid measuring 4. We then have constructed a parallelogram whose altitude is 3 and whose upper and lower bases are respectively 5 and finding the area in the regular way we would have $5 \times 3 = 15$ directly for area of the trapezoid.

$$\frac{\left(\begin{matrix}\textit{Sum of bases}\\ 6 + 4\end{matrix}\right) \times \begin{matrix}\textit{Alt.}\\ \times\ 3\end{matrix}}{2} = \text{Area.} \qquad \frac{\overset{5}{\not 10} \times 3}{\not 2} = 15 \text{ Area.}$$

A trapezoid may be divided into two triangles; and its area will be the sum of the area of the two triangles. As the bases of the triangles are the parallel sides of the trapezoid, we have,

$$\frac{\text{B} \times \text{Alt}}{2} + \frac{\text{B} \times \text{Alt}}{2} \text{ or } \frac{(\text{Sum of Parallel sides}) \times \text{Alt}}{2} = \text{Area.}$$

A trapezoid has bases 20 and 15 and an altitude of 10. What is its area?

$$\frac{20 \times 10}{2} + \frac{15 \times 10}{2} \text{ or } \frac{(15 + 20) \times 10}{2} = ?$$

$$\left(\frac{\overset{5}{\not 20} \times \not 10}{\not 2} = 100\right) + \left(\frac{\overset{5}{15} \times \not 10}{\not 2} = 75\right) \text{ or } \frac{\overset{35}{(\not 15 + 20)} \times \overset{5}{\not 10}}{\not 2} = 175$$

$$100 + 75 = 175$$

3. **Trapezium**—quadrilateral with none of the sides parallel.

a. **Rule for Area**—Divide it into two triangles and add the area of each triangle.

F. **Regular polygons** of more than four sides may be divided into equal triangles. The sum of the bases of the triangle would be equal to the perimeter of the polygon; and the altitude of the triangles would be equal to the apothem of the polygon, hence;

a. **Rule**—Perimeter $\times$ Apothem = Area Polygon.

Examples. Parallelograms.

1. **Square.**

a. What is the area of a field measuring 40 rds. on a side?

B $\times$ Alt. = Area. $\qquad$ rds. rds. $\quad 40 \times 40 = 1600$ rds. Answer.

(Measuring 40 rds. on a side indicates all sides are equal, hence we have a square.)

b. If the area of a square field is 1600 square rods, what is its length?

B $\times$ Alt. = 1600 sq. rds. $\qquad$? $\times$? = 1600 sq. rds.

Two terms missing, we cannot use the formula; but we know a square is the product of two equal factors, and, having the square, we extract the root to find either factor.

$$\sqrt{1600 \text{ sq. rods}} = 40 \text{ rds. either base or altitude.}$$

2. **Rectangle.**

a. What is the area of a rectangular field whose length is 60 rods and whose width is 40 rods.

B $\times$ Alt. = Area. 60 rds. $\times$ 40 rds. = 2400 sq. rods.

b. What is the length of the field whose width is 40 rods and whose area is 2400 square rods.

B $\times$ Alt. = Area. ? $\times$ 40 rods = 2400 sq. rods. 2400 $\div$ 60 = 40 rods.

3. **Rhombus**—How many square rods in a field in the shape of a rhombus whose base is 10 rds. and the altitude 8 rds. ?

B $\times$ Alt. = Area. 10 rds. $\times$ 8 rds. = 80 sq. rds.

Use the same formula for any missing term, same as in Examples under Rectangle.

4. **Rhomboid**—How many square rods in a field in shape of a rhomboid, base 8 rods, altitude 6 rods? B $\times$ Alt. = Area. $8 \times 6 = 48$ square rods.

5. **Trapezoid**—How many square feet in a board 18 feet long, 15 inches wide at one end and 21 inches wide at the other?

$$\frac{\text{Sum of parallel sides} \times \text{Alt.}}{2} = \text{Area.} \qquad \frac{\left(\frac{21}{3} + \frac{15}{3}\right) \times 18 \text{ ft.}}{2} = ?$$

$$\frac{12 \times \overset{9}{\cancel{18}}}{\cancel{2}} = 108 \text{ square feet.} \quad \text{Answer}$$

6. **Trapezium.**

a. How many square feet in a surface the length of whose sides are respectively 30, 40, 35, and 45 feet, and the diagonal 50 feet? Answer 1161.2 square feet.

Divide the trapezium into two triangles, the three sides being given in each. Find the area for each by Rule 2 under triangles and add results.

b. Find the area of a trapezium whose diagonal is 35 feet and whose perpendiculars to the diagonal are 8 feet and 12 feet. Answer 350 square feet.

Divide the trapezium into two triangles, having the base and altitude of each given. Find the area of each by Rule 2 under triangles and add the results.

7. **Circle**—is a plane figure bounded by a curved line, every point of which is at an equal distance from a central point within.

1. **Circumference**—is the curved line bounding the circle.

2. **Diameter**—is a straight line passing through the center from a point in the circumference to another point in the circumference.

3. **Radius**—is a straight line from the center to a point in the circumference. It equals $\frac{1}{2}$ the diameter.

π Pronounced (pi) is a Greek letter which stands for 3.1416. It indicates the relation between the circumference and the diameter of a circle. It has been proved by geometry that the circumference is about 3.1416 times, or as is now generally taken $3\frac{1}{7}$ times the diameter. Measuring *around* and *across* a circular object the circumference is about $3\frac{1}{7}$ times the diameter. Therefore we have the

4. **Rule**—Diameter $\times$ $3\frac{1}{7}$ = Circumference.

Example 1. What is the circumference of a circle whose radius is 12 inches?

$D \times \frac{22}{7} =$ Circumference, but Radius $\times 2 =$ Diameter.

$\left(28 \times 2\right) \times \frac{22}{7} =$ Circumference. $\overset{8}{\cancel{56}} \times \frac{22}{\cancel{7}} = 176$ in. Answer.

Example 2. What is the diameter of a circle whose circumference is 176 in.?

$D \times \frac{22}{7} =$ Circumference. $? \times \frac{22}{7} = 176.$

$176 \div \frac{22}{7} = \overset{8}{\cancel{176}} \times \frac{7}{\cancel{22}} = 56$ in. Answer.

If the radius is required, divide the diameter by 2, making 28 inches.

5. **Area of Circle.**

A circle may be divided into many equal figures which are nearly triangles—the altitude of each being equal to the radius of the circle; and the sum of the bases being equal to the circumference.

The area of a triangle we know is $\frac{B \times \text{Alt.}}{2} =$ Area.

Using the names of the circle equivalent to these, we have

Rule 1. $\frac{\textit{Circumference} \times \textit{Radius}}{2} =$ Area of Circle.

We also know that Diameter $\times \frac{22}{7} =$ Circumference,

or if radius is given 2 times the radius gives the diameter, so

2 Radius $\times \frac{22}{7} =$ Circumference.

Substituting in equation (1) for circumference its value, we have

$\frac{\left(2 \text{ Radius} \times \frac{22}{7}\right) \times \text{Radius}}{2} =$ Area; cancelling and simplifying gives,

Rule 2. $\text{Radius}^2 \times \frac{22}{7} =$ Area of Circle.

Examples.

1. Find the area of a circle whose diameter is 10 ft. and the circumference 32 ft.

$\frac{C \times R}{2} =$ Area. $D = 10$ $R = \frac{10}{2}$

$\frac{32 \times \frac{10}{2}}{2} =$ Area. $\overset{16}{\cancel{32}} \times \frac{\overset{5}{\cancel{10}}}{\cancel{2}} \times \frac{1}{\cancel{2}} = 80$ sq. ft. Answer.

2. Find the area of a circle whose diameter is 10 feet.

$\text{Radius}^2 \times \frac{22}{7} =$ Area. $\frac{\overset{5}{\cancel{10}}}{\cancel{2}} \times \frac{\overset{5}{\cancel{10}}}{\cancel{2}} \times \frac{22}{7} = \frac{550}{7} = 78\frac{4}{7}$ sq. ft. Answer.

3. How much land in a circular garden having an 80-rod fence surrounding it?

$D \times \frac{22}{7} =$ Cir. $? \times \frac{22}{7} = 80$ rds.

80 rds. $\div \frac{22}{7} = \overset{40}{\cancel{80}} \times \frac{7}{\cancel{22}} = \frac{280}{11} = 25\frac{5}{11}$ rds. Diameter.

$R^2 \times \frac{22}{7} =$ Area. $(25\frac{5}{11} \div 2)^2 \times \frac{22}{7} = ?$

$\frac{280}{11} \times \frac{1}{2} \times \frac{\overset{20}{\cancel{140}}}{\cancel{280}} \times \frac{1}{2} \times \frac{\overset{2}{\cancel{22}}}{7} = \frac{5600}{11} = 509\frac{1}{11}$ sq. rds. Area. Answer.

4. What is the circumference of a circle having an area of $509\frac{1}{11}$ square rods.

$$R^2 \times \frac{22}{7} = \text{Area.} \qquad ? \times \frac{22}{7} = 509\frac{1}{11} \text{ square rods.}$$

$$509\frac{1}{11} \div \frac{22}{7} = \frac{\overset{2800}{\cancel{5600}}}{11} \times \frac{7}{\underset{11}{\cancel{22}}} = \frac{19600}{121} = 161.1570 \text{ R}^2 \qquad \sqrt{161.1570} = 12.69 \text{ rds. Radius}$$

$$C \times \frac{R}{2} = \text{Area.} \qquad ? \times \frac{12.69}{2} = 509\frac{1}{11} \text{ square rods.}$$

$$509\frac{1}{11} \div \frac{12.69}{2} = \frac{5600}{11} \times \frac{2}{12.69} = \frac{11200}{139.59} = 80 \text{ rods; or,}$$

$$D \times \frac{22}{7} = \text{Cir.} \qquad 25.38 \times \frac{22}{7} = \frac{358.36}{7} = 79.75 \text{ rods. Answer.}$$

5. What is the diameter of a circle whose area is $509\frac{1}{11}$ square rods?

$$R^2 \times \frac{22}{7} = \text{Area.} \qquad ? \times \frac{22}{7} = 509\frac{1}{11} \text{ square rods.}$$

$$509\frac{1}{11} \div \frac{22}{7} = \frac{\overset{2800}{\cancel{5600}}}{11} \times \frac{7}{\underset{11}{\cancel{22}}} = \frac{19600}{121} = 161.1570 \text{ R}^2. \qquad \sqrt{161.1570} = 12.69 \text{ rds. Radius}$$

$$R^2 \times 2 = \text{Dia. } 12.69 \times 2 = 25.38 \text{ rods. Answer.}$$

G. **Solids**—A solid is anything with three dimensions, length, breadth, and thickness.

1. **Prism** is a solid whose ends are equal parallel polygons and whose sides are parallelograms.
   a. **Kinds of Prisms**—they take their names from the shapes of their bases; as, *Triangular*, *Square*, *Hexagonal*, *Rectangular*, etc.
   b. **Altitude** is the perpendicular distance between the bases.
   c. **Convex Surface** is all the parallelograms making up the sides.
   d. **Entire Surface** is the convex surface plus the area of the two bases.

(Fold paper about a prism and then spread it out showing that the convex surface of the prism is really a rectangle in shape.)

2 **Cylinder** is a solid bounded by a curved surface and one whose bases are equal, parallel circles.
   a. **Altitude** and **Surface** correspond to the same terms in the prism.

Rule—Area of Prism or Cylinder:
1. Perimeter Base $\times$ Altitude = *Convex Surface.*
2. Convex Surface + Area of 2 Bases = *Entire Surface.*

3. **Pyramid** is a solid whose sides are triangles meeting at a point called the vertex and has one base which is a polygon.
   a. **Kinds of Pyramids**—they take their names from the shapes of their bases; as, *Triangular*, *Hexagonal*, etc.
   b. **Altitude**—perpendicular distance between the vertex and the base.
   c. **Slant Height**—altitude of one of the triangles making up the sides of the pyramid.
   d. **Convex Surface**—all the triangles making up the sides.
   e. **Entire Surface**—convex surface plus the area of the base.

4. **Cone** is a solid bounded by a curved surface tapering to a point called the apex, and one which has a circle as its base.

a. **Altitude—slant height—surface** correspond to the same terms in the pyramid.

b. **Convex surface of pyramid and cone.**

Rule $\frac{\text{Perimeter Base} \times \text{Slant Height}}{2} = \text{Convex Surface.}$

Volume—the contents of any solid, expressed in cubic units.

Volume of Prism or Cylinder. Rule—Area of Base $\times$ Altitude = Volume.

Volume of Pyramid or Cone. Rule $\frac{\text{Area of Base} \times \text{Alt.}}{3} = \text{Volume.}$

It has been proved that if a hollow pyramid and prism having the same base and altitude be filled with a substance it will take 3 full pyramids to make the same quantity contained in the prism—the same is true of a cone and cylinder of the same altitude and base.

As the volume of a prism or a cylinder is found by multiplying the area of the base by the altitude, and if the pyramid or the cone is just a third of their size, if we divide by 3 we will have the *Area of the Pyramid or the Cone.*

Example 1. What is the *convex surface* of a *square prism* whose base is 2 feet on a side and whose altitude is 4 feet. What is its *entire surface?*

Perimeter of Base $\times$ Altitude = Convex Surface.

$(2 \times 4) \times 4 = 32$ square feet.

(Being a square prism the base has 4 sides, all equal, so the perimeter is 2×4.)

Convex Surface + Area 2 Bases = Entire Surface.

$32 + 2\ (4 \times 4) = ?$ $\qquad 32 + 32 = 64$ square feet. Entire Surface.

2. What is the *convex surface* of a *cylinder* whose diameter is 2 feet and whose altitude is 4 feet. What is the *entire surface?*

Perimeter Base $\times$ Altitude = Convex Surface.

$$\left(2 \times \frac{22}{7}\right) \times 4 = \frac{176}{7} = 25\tfrac{1}{7} \text{ square feet.}$$

(Perimeter of base is the circumference of the circle and that is found by using the formula $D \times \frac{22}{7} = \text{Circumference}$, so $2 \times \frac{22}{7} = \text{Perimeter Base.}$)

Base is a circle so $\frac{R^2 \times 22}{7} = \text{Area.}$ $\qquad \frac{2}{2} \times \frac{2}{2} \times \frac{22}{7} = 3\tfrac{1}{7}$ sq. ft. Answer.

Convex Surface + Area 2 Bases = Entire Area. $\qquad 25\tfrac{1}{7} + 6\tfrac{2}{7} = 31\tfrac{3}{7}$ sq. ft. Ans.

3. What is the *convex surface* of a *square pyramid* whose base is 4 feet and whose slant height is 6 feet. What is the *entire surface?*

$\frac{\text{Perimeter Base} \times \text{Slant Height}}{2} = \text{Convex Surface.}$ $\qquad \frac{(4 \times 4) \times \overset{3}{\cancel{6}}}{\underset{1}{\cancel{2}}} = 48$ sq. ft.

B $\times$ Alt. = Area Base. $\qquad 4 \times 4 = 16$

Convex Surface + Area Base = Entire Surface. $\qquad 48 + 16 = 64$ sq. ft.

4. What is the *convex surface* of a *cone* whose slant height is 6 feet and whose diameter is 2 feet?

$$\frac{\text{Perimeter Base} \times \text{Slant Height}}{2} = \text{Convex Surface.}$$

Perimeter of Base is the circumference of a circle,

and as $D \times \frac{22}{7} = \text{Circumference}$, we have

$\frac{2 \times \frac{22}{7} \times 6}{2} = \text{Convex Surface.}$ $\qquad \underset{1}{\cancel{2}} \times \frac{22}{7} \times 6 \times \frac{1}{\underset{1}{\cancel{2}}} = \frac{132}{7} = 18\tfrac{6}{7}$ sq. ft.

5. *Cone* has a convex surface of $18\frac{6}{7}$ square feet and the radius of the base is 1 foot. What is the *slant height?*

$$\frac{\text{Perimeter Base} \times \text{Slant Height}}{2} = \text{Convex Surface.} \qquad \frac{2 \times \frac{22}{7} \times ?}{2} = 18\tfrac{6}{7} \text{ sq. ft.}$$

$$2 \times \frac{22}{7} \times \frac{1}{2} \times ? = 18\tfrac{6}{7} \qquad 18\tfrac{6}{7} \div \left(2 \times \frac{22}{7} \times \frac{1}{2}\right) = ?$$

$$\frac{\overset{6}{\cancel{132}}}{\underset{1}{\cancel{7}}} \times \frac{1}{\cancel{2}} \times \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{22}}} \times \frac{\cancel{2}}{\underset{1}{\cancel{1}}} = 6 \text{ ft.} \quad \text{Answer.}$$

6. What is the *volume* of a *cylinder* whose altitude is 4 feet and whose diameter is 2 feet?

Area of Base $\times$ Alt. = Volume. Dia. 2 ft. = 1 ft. Radius.

$$R^2 \times \frac{22}{7} = \text{Area Base.}$$

$$\frac{\cancel{2}}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{2}} \times \frac{22}{7} \times 4 = \frac{88}{7} = 12\tfrac{4}{7} \text{ cu. ft.} \quad \text{Answer.}$$

7. A *square pyramid* has a lateral surface of 48 square feet and the slant height is 6 feet. What is the *perimeter* of the *base?* Length of one side of base?

$$\frac{\text{Perimeter Base} \times \text{Slant Height}}{2} = \text{Convex Surface.} \qquad \frac{? \times \overset{3}{\cancel{6}}}{\cancel{2}} = 48$$

$48 \div 3 = 16$ ft. Perimeter of base. Ans. $\frac{16}{4} = 4$, length of one side of base. Answer.

8. A *prism* has a square base and a volume of 24 cubic feet and its height is 6 feet. What is the *length* of *each side* of the *base?*

Area Base $\times$ Alt. = Volume. $? \times 6 = 24$

cu.ft. ft. sq.ft.
$24 \div 6 = 4$ sq. ft., area of base. Base being a square $\sqrt{4} = 2$ feet. Length of each side.

9. Find the *contents* of a *cone* whose diameter is 2 feet and whose altitude is 4 feet.

$$\frac{\text{Area Base} \times \text{Altitude}}{3} = \text{Volume.} \qquad \frac{\cancel{2}}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{2}} \times \frac{22}{7} \times \frac{4}{3} = \frac{88}{21} = 4\tfrac{4}{21} \text{ cu. ft.} \quad \text{Ans.}$$

10. Find the *contents* of a *square pyramid* 2 feet on side and 6 feet altitude.

$$\frac{\text{Area Base} \times \text{Alt.}}{3} = \text{Volume.} \qquad \frac{2 \times 2 \times \overset{2}{\cancel{6}}}{\cancel{3}} = 8 \text{ cu. ft.} \quad \text{Answer.}$$

5. **Sphere** is a solid bounded by a curved surface, every point of which is an equal distance from the center.

a. **Diameter** is a line passing through the center and connecting two points in the surface of the sphere.

b. **Radius** is half the diameter.

c. **Convex Surface**—By geometry it has been proved that the surface of a sphere is equal to the area of 4 circles whose radii are the same as that of the sphere.

$$R^2 \times 3\tfrac{1}{7} = \text{Area Circle.}$$

Rule 1. $4\,(R^2 \times 3\tfrac{1}{7}) =$ Area of 4 Circles or Surface of Sphere.

$D \div 2 = R.$ $\quad 4 \times \frac{D}{2} \times \frac{D}{2} \times 3\tfrac{1}{7} =$ surface of sphere, or cancelling and reducing

Rule 2. $D^2 \times 3\tfrac{1}{7} =$ Surface Sphere.

d. **Volume Sphere**—By geometry we find that $\frac{1}{3}$ of (Radius times the surface) = Volume of a Sphere.

Sphere could be cut into innumerable pyramids, the altitude of each pyramid being

the same as the radius of the sphere, hence the sum of the bases of all these pyramids would be the surface of the sphere.

The volume of a pyramid is found by $\frac{\text{Area of Base} \times \text{Alt.}}{3}$

Hence, substituting the other names, we have $\frac{\text{Convex Surface} \times \text{Radius}}{3} = \text{Volume}$ and convex surface was found by formula $4\ R^2 \times 3\frac{1}{7}$, so substituting that we have $\frac{4\ R^2 \times 3\frac{1}{7} \times R}{3} = \text{Volume}$, simplified gives $\frac{4}{3} R^3 \times 3\frac{1}{7} = \text{Volume}$, but $D \div 2 = R$. $\therefore$ $\frac{4}{3} \times \frac{D}{2} \times \frac{D}{2} \times \frac{D}{2} \times 3\frac{1}{7} = \frac{1}{6} \times D^3 \times 3\frac{1}{7} = \text{Volume}$, or $\frac{D^3}{6} \times 3\frac{1}{7} = \text{Volume}$.

Example 1. What is the *surface* of a *sphere* whose diameter is 4 inches?

$D^2 \times 3\frac{1}{7} = \text{Surface Sphere.}$ $(4 \times 4) \times \frac{22}{7} = \frac{176}{7} = 25\frac{1}{7}$ sq. in. Answer.

2. What is the *volume* of a *sphere* whose diameter is 4 inches?

$\frac{1}{6} \times D^3 \times 3\frac{1}{7} = \text{Volume.}$ $\frac{1}{3} \times \overset{2}{4} \times 4 \times 4 \times \frac{22}{7} = \frac{352}{21} = 25\frac{7}{21}$ cu. in. Ans.

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